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Small Volume Limits of 2-*d* Yang–Mills

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Abstract. By examining the lattice gauge approximation we show that the small volume limit of the 2-dimensional Yang–Mills functional integral is the natural symplectic measure on the moduli space of flat connections.

0. Introduction

The subject of this paper is the small volume limit of the 2-dimensional Yang–Mills functional integral. By examining the lattice gauge approximation, we show that this limit is precisely the natural symplectic measure on the moduli space of flat connections. This answers affirmatively a question raised in [S2].

We begin by placing this result in context. Let Σ be a compact orientable surface of genus $g > 1$, G a compact Lie group with finite center, and $P \to \Sigma$ a principal G-bundle. We suppose further that Σ is endowed with a volume form ε, and G is equipped with a bi-invariant Riemannian metric with total volume 1. For any connection A on P, we associate the curvature F_A. (See [A-B] for details). We have

$$F_A = f_A \varepsilon$$

for some ad(P) valued function f_A. The Yang–Mills functional is defined by

$$\mathrm{YM}_\varepsilon(A) = \int_\Sigma \| f_A \|^2 \varepsilon \, .$$

Let $\mathscr{A}(P)$ be the affine space of connections on P. We are interested in the partition function

$$Z(\Sigma, \varepsilon, k, P) = \int_{\mathscr{A}(P)} \mathscr{D}A e^{-\frac{1}{k^2}\mathrm{YM}_\varepsilon(A)} \, ,$$

where k^2 is a coupling constant. More precisely, the object of interest is

$$Z(\Sigma, \varepsilon, k) = \sum_P Z(\Sigma, \varepsilon, k, P) \, ,$$