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Classification of the Indecomposable Bounded Admissible Modules over the Virasoro Lie Algebra with Weightspaces of Dimension not Exceeding Two

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Abstract. In view of [1, 2] any bounded admissible module $\mathscr{A}$ over the Virasoro Lie algebra $\mathscr{V}$ is a finite length extension of irreducible modules with one-dimensional weightspaces. To each extension of finite length n are associated $n + 1$ invariants $(a_1, \Lambda_1, \ldots, \Lambda_n)$. We prove that we have $\Lambda_i - \Lambda_j \in \{0, 1, \ldots 6(n-1)\}$ for all (i, j) with $1 \leqq i \leqq j \leqq n$. In the case $n = 2$ this result allows us to construct all the indecomposable bounded admissible $\mathscr{V}$ modules, where the dimensions of the weightspaces are less than or equal to two. In particular we obtain all the extensions of two irreducible bounded $\mathscr{V}$-modules.

I. Introduction

The Virasoro algebra $\mathscr{V}$ is the complex Lie algebra with basis $\{C, x_n, n \in \mathbb{Z}\}$ and commutation relations:

$$[x_i, x_j] = (j - i)x_{i+j} + \delta_{i,-j}\frac{j^3 - j}{12}C \quad \forall i, \forall j \in Z ,$$

$$[C, x_i] = 0 .$$

We set also $Q_1 = -x_1 x_{-1} + x_0^2 - x_0$.

A $\mathscr{V}$-module is said to be admissible if it satisfies the two conditions:

a) x_0 acts semi-simply.

b) The eigenspaces of x_0 (also called weight-spaces) are finite-dimensional.

Recently, the classification of irreducible admissible $\mathscr{V}$-modules has been achieved in [1, 2]. Besides the highest or lowest weight $\mathscr{V}$-modules, it furnishes a second class of $\mathscr{V}$-modules where the weightspaces are one-dimensional. These latter are the following:

– The $\mathscr{V}$-modules of Feigin–Fuchs $A(a, \Lambda)$ with $(a, \Lambda) \in \mathbb{C}^2$ and $0 \leqq \operatorname{Re} a < 1$ ($a = 0 \Rightarrow \Lambda \neq 0, 1$), whose action is given on a basis $\{v_n, n \in \mathbb{Z}\}$ by:

$$x_i v_n = (a + n + i\Lambda)v_{n+i} \quad Cv_n = 0 \quad \forall n, \forall i . \tag{I.1}$$