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A Nonintegrable Model in General Relativity

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Abstract. The geodesic flow of a perturbation of the Schwarzschild metric is shown to possess a chaotic invariant set. The perturbed metric is a relativistic analogue of Hill's problem in classical celestial mechanics in that it models the effects of a distant third body.

1. Introduction

This paper is concerned with a dynamical system which models the motion of a small mass near a black hole but also subject to the gravitational effects of a third, more distant mass. It is shown that under certain conditions, the system posesses hyperbolic periodic orbits with transverse homoclinic orbits. The resulting chaotic dynamics contrasts sharply with the behavior of the better-known general relativistic systems, which have turned out to be completely integrable.

In classical celestial mechanics, the motion of a small point mass around a large mass is modelled by the Kepler problem. The effects of a third mass could be treated as a perturbation. For example, a perturbing potential due to Hill is sometimes added to describe the effects of a distant third mass [Hill]. The Kepler problem is completely integrable. In fact, it possesses additional constants of motion beyond those required for integrability with the result that all of the bounded orbits are periodic rather than merely quasi-periodic as is typical for integrable systems. However, integrable systems are rare and delicate. One expects that most perturbations will destroy the integrability. This is difficult to show for perturbations of the Kepler problem because the unperturbed problem is so degenerate.

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