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Coherent States and Geometric Quantization*

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Abstract. Based on the concept of generalized coherent states, a theory of mechanical systems is formulated in a way which naturally exhibits the mutual relation of classical and quantum aspects of physical phenomena.

1. Introduction

In this work we present a description of physical systems which, in a sense, unifies into one theory the formalisms of both quantum and classical mechanics. The construction is based on the conviction that all experimentally achievable states of any physical system are parameterized with help of an appropriate finite-dimensional manifold M. Simultaneously, as a basic object of both experiment and theory, a transition amplitude is chosen. This is to be interpreted as a probability amplitude for a system in the state parameterized by $q \in M$ to be in the state parameterized by $p \in M$. One assumes, in accordance with well established experience, that the transition amplitude possesses some natural properties. These are the properties which allow one to construct a map $K: M \to \mathbb{CP}(\mathcal{M})$ of the manifold M into the complex projective Hilbert space $\mathbb{CP}(\mathcal{M})$. The Hilbert space $\mathcal{M}$ and the map K are uniquely defined by the transition amplitude. And vice versa, once the map $K: M \to \mathbb{CP}(\mathcal{M})$ is given, the transition amplitude for a system can be recovered.

In the paper we limit ourselves to the case in which K is a symplectic embedding. This means that the pull-back $K^*\omega_{FS}$ of the Fubini-Study form ω_{FS} of $\mathbb{CP}(\mathcal{M})$ is again a symplectic form. We define a mechanical system as a triple $(M, \mathcal{M}, K: M \to \mathbb{CP}(\mathcal{M}))$, where $(M, K^*\omega_{FS})$ is interpreted as the phase space of classical states of the system, while $(\mathbb{CP}(\mathcal{M}), \omega_{FS})$ is the phase space of its pure quantum states. M might be considered to represent a family of all admissible results of measurements with classical devices used to measure parameters

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