

Sharp Estimates for Dirichlet Eigenfunctions in Horn-Shaped Regions

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Abstract. We prove sharp exponential decay for the Dirichlet eigenfunctions in horn-shaped regions in the plane. The estimate is obtained using a method of Carleman which is widely used in the study of harmonic measure. Such estimates can be applied to study the intrinsic ultracontractivity properties of the heat semigroup for such regions.

0. Introduction

The purpose of this paper is to prove sharp exponential bounds for the decay of the Dirichlet eigenfunctions in “infinite trumpets” or “horn-shaped” regions. Such estimates, besides being of interest in their own right, can be used to prove *intrinsic ultracontractivity*, IU, for the Dirichlet Laplacian for these regions as in Davies and Simon [4]. Also, our result sharpens Theorem 7.3 in Davies and Simon [4].

Let $\theta: (0, \infty) \rightarrow [0, 1]$ be continuous and let

$$D_\theta = \{z = (x, y) : x > 0, -\theta(x) < y < \theta(x)\}. \tag{0.1}$$

It was proved in Davies and Simon [4] that under the assumptions (i) $\theta'(x)$ bounded and C^1 , (ii) $\theta(x) \rightarrow 0$ as $x \rightarrow \infty$, (iii) $\theta'(x) \leq 0$ for large x and (iv) $\theta'(x)/\theta(x) \rightarrow 0$ as $x \rightarrow \infty$, there exist constants C_1, C_2, D_1, D_2 such that

$$C_1 \exp\left(-\frac{D_1 x}{\theta(x)}\right) \leq \varphi(x, 0) \leq C_2 \exp\left(-\frac{D_2 x}{\theta(x)}\right), \tag{0.2}$$

where φ is the positive eigenfunction of D_θ . Our result, which was motivated by (0.2), is

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