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Local Rules for Quasiperiodic Tilings of Quadratic 2-Planes in R^4

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Abstract. We prove that quasiperiodic tilings of the plane, appearing in the strip projection method always admit local rules, when the linear embedding of $\mathbb{R}^2$ in $\mathbb{R}^4$ has quadratic coefficients. These local rules are constructed and studied. The connection between Novikov quasicrystallographic groups and the quasiperiodic tilings of Euclidean space is explained. All the point groups in Novikov's sense, compatible with these local rules, are enlisted. The two-dimensional quasicrystals with infinite-fold rotational symmetry are constructed and studied.

Introduction

Quasicrystals (QC) are the quasiperiodic tilings of the Euclid space $\mathbb{R}^k$ by a finite (up to translations) number of polyhedra. For the history and reviews we refer to [2–7]. By now, several approaches have been suggested.

One of them, initiated by S. P. Novikov in 1986, is based upon the following definition of the quasicrystallographic group:

Definition. We shall call a finite-generated abelian subgroup $T \subset \mathbb{R}^k$, which generates $\mathbb{R}^k$ as a linear space the quasilattice in $\mathbb{R}^k$.

Definition. A subgroup G of the group E_k of all isometries of k-dimensional Euclid space is called a *k-dimensional quasicrystallographic group*, iff its intersection with the subgroup $\mathbb{R}^k \subset E_k$ of all translations is some quasilattice $T \subset \mathbb{R}^k$.

Definition. The above defined quasilattice $T \subset \mathbb{R}^k$ is called *the subgroup of translations of the quasicrystallographic group G*, and the factor-group $R = G/T$ is called *the point group*, or *the group of orthogonal parts of the quasicrystallographic group G*.

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