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Classical States and the BRST Charge

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Abstract. The role played by the BRST-charge in isolating the physical states in a classical first-class constrained system is analysed. Contrary to popular belief, the cohomological argument used to characterize the physical observables in such a system does not extend to the classical states. It is shown that, in order to recover the physical states, the BRST-charge must be augmented with a new charge, of ghost number minus one, constructed out of a set of gauge fixing conditions for the original constraints. The relevance of this construction to the quantum theory is discussed.

1. Introduction

In recent years the use of ghost variables has been extended from a diagrammatic trick to maintain unitarity in one-loop calculations [1], to a general procedure for isolating the physical observables in both quantum and classical first-class constrained systems [2–7]. Although assigning ghost variables a classical role may, at first sight, seem rather surprising, their use in classical dynamics can be given a precise mathematical meaning which, in turn, supplies an important theoretical underpinning to their applications in the quantum theory.

The aim of this paper is to use such a classical analysis to investigate the role of ghost variables in directly isolating the physical states of a constrained system (all constraints in this paper will be first-class). This is motivated by the observations [8–10] that in the quantum theory the *natural* definition of physical states (ghost number zero states that are BRST invariant but not the BRST transform of another state) does not yield a satisfactory result (indeed, such states generically have zero norm). We shall show that this apparent complication should come as no surprise since it is also there in the classical theory. We shall also see that there is a straightforward solution to this problem of isolating the classical physical states that can be applied directly to the quantum theory.

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