Commun. Math. Phys. 148, 359–376 (1992)

Communications in Mathematical Physics

The Extended Bethe Ansatz for Infinite $S=1/2$ Quantum Spin Chains with Non-Nearest-Neighbor Interaction

V. I. Inozemtsev

International Centre for Theoretical Physics, I-34100 Trieste, Italy and
Laboratory of Theor. Phys., JINR, Head Post Office P.O. Box 79, Moscow (Dubna), Russia*

Received September 25, 1991

Abstract. It is shown that the description of the states of infinite $S=1/2$ interacting spin systems with the Hamiltonian $H_s=-\frac{J}{2}\sum_{j\neq l,\, j,l\in\mathbf{Z}} a^2\sinh^{-2}a(j-l)\frac{(\sigma_j\sigma_l-1)}{2}$ can be performed by studying the hyperbolic Calogero-Sutherland eigenvalue problem. The construction of multimagnon wave functions in each N-magnon sector is based on solutions of the set of linear algebraic equations which determine also the structure of zonal spherical functions on symmetric spaces $X_N^- = SL(N,\mathbf{H})/\mathrm{Sp}(N)$ of negative curvature. The usual Bethe Ansatz for the XXX Heisenberg model corresponds to asymptotic forms of these wave functions at small values of a^{-1} or large distances between spins turned over the ferromagnetic ground state.

1. Introduction

Starting with the paper of Bethe [1], the investigation of one-dimensional exactly solvable models of interacting objects (spins, classical or quantum particles in the schemes of first and second quantization) has given a number of results both of physical and mathematical significance. One of the highlights in this branch of mathematical physics is the Yang-Baxter equation [2, 3] which serves as a source of continuous development in the study of various aspects of group theory and low-dimensional statistical mechanics. Most of the well-known statistical models both in one- and two-dimensional cases have solutions in the form of the Bethe Ansatz in its classical [1, 2, 4] or algebraic [5] versions with some more or less sophisticated modifications. On the other hand, there is a family of systems which were proved to be completely integrable [6–11], but the solutions were not included into the Bethe Ansatz [9–10, 16] or still remained unknown.

* Permanent address