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On the Selection Rules for Spin-Lorentz Cobordisms

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Abstract. A recent result of Gibbons and Hawking on the existence of spin-Lorentz cobordisms is applied to the class of 3-manifolds that can support a non-negative scalar curvature metric. We call such manifolds admissible. It is shown how in general the existence of a spin-Lorentz cobordism restricts the number-mod-2 of certain prime manifolds to occur in the prime decomposition, which are explicitly listed in the case of admissible manifolds.

Introduction

Very recently, Gibbons and Hawking derived a necessary and sufficient condition for a closed, orientable 3-manifold, Σ, to spin-Lorentz bound, that is, to be the spacelike boundary of a Lorentzian 4-Manifold, M, that admits a $SL(2, C)$-spin structure [2]. Their result was that the mod-2 Kervaire semi-characteristic, $\mathcal{U}(\Sigma)$, must be zero. Here, $\mathcal{U}(\Sigma)$ can itself be defined through the combination of cohomology groups:

$$\mathcal{S}(\Sigma) = \dim H^0(\Sigma, Z_2) + \dim H^1(\Sigma, Z_2), \tag{1}$$

$$\mathcal{U}(\Sigma) = \mathcal{S}(\Sigma) \bmod 2. \tag{2}$$

In Sect. 1 we want to reformulate this expression in terms of homology groups with integer coefficients and use it to derive a simple behaviour of $\mathcal{U}(\Sigma)$ under taking disjoint and connected sums. This is summarized in Corollary 1. In Sect. 2 we restrict attention to the so-called *admissible* 3-manifolds, which are characterized by admitting a metric of non-negative scalar curvature. Assuming a standard conjecture in 3-manifold theory to hold, we can give an explicit list of those prime manifolds whose occurrence is restricted mod 2 if their connected sum is required to spin-Lorentz bound, or if required to be spin-Lorentz cobordant to

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