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The Spinor Heat Kernel in Maximally Symmetric Spaces

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Abstract. The heat kernel $K(x, x', t)$ of the iterated Dirac operator on an N-dimensional simply connected maximally symmetric Riemannian manifold is calculated. On the odd-dimensional hyperbolic spaces K is a Minakshisundaram–DeWitt expansion which terminates to the coefficient $a_{(N-1)/2}$ and is exact. On the odd spheres the heat kernel may be written as an image sum of WKB kernels, each term corresponding to a classical path (geodesic). In the even dimensional case the WKB approximation is not exact, but a closed form of K is derived both in terms of (spherical) eigenfunctions and of a "sum over classical paths." The spinor Plancherel measure $\mu(\lambda)$ and ζ function in the hyperbolic case are also calculated. A simple relation between the analytic structure of μ on H^N and the degeneracies of the Dirac operator on S^N is found.

1. Introduction

A maximally symmetric Riemannian manifold M of dimension N has an isometry group of maximum dimension $N(N+1)/2$. M is also a constant curvature space, i.e., the Riemann tensor takes the form

$$R_{abcd} = k(g_{ad}\, g_{bc} - g_{ac}\, g_{bd}) , \tag{1.1}$$

where k is a constant. The Ricci tensor and curvature scalar are given by $R_{ad} = k(N-1)g_{ad}$, and $R = kN(N-1)$. Moreover, M is necessarily isometric to one of the following spaces: a) Euclidean space $R^N (k = 0)$; b) the sphere S^N of radius a $(k = 1/a^2)$; c) the real projective space $P^N(R) = S^N/\sim$, where $\sim$ is the antipodal points identification (k is the same as for S^N); d) the real hyperbolic space $H^N(R)$ of radius a $(k = -1/a^2)$ (see ref. [24], vol. 1, p. 308). These spaces are all simply connected except for $P^N(R)$, which is doubly connected.

In the pseudo-Riemannian Lorentzian case [signature $(-, +, \ldots, +)$] we have, similarly, that the maximally symmetric spacetimes are Minkowski spacetime M^N (zero curvature), de Sitter spacetime $(\mathrm{dS})_N$ (positive curvature), and anti-de Sitter spacetime $(\mathrm{AdS})_N$ (negative curvature). In the Euclidean approach to