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Irrational Free Field Resolutions for $W(sl(n))$ and Extended Sugawara Construction

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Abstract. The existence of Miura-type free field realizations is established for the extended conformal algebras $W(sl(n))$ at irrational values of the screening parameter. The problem of the "closure" of the algebra is reduced to a finite dimensional quantum group problem. The structure of the Fock space resolution and the character formula are obtained for the irreducible modules. As graded vector spaces these modules are shown to be isomorphic to the space of $sl(n)$ singlets in $\widehat{sl}(n)$ affine level 1 modules. The isomorphism is given by the $\phi\beta\gamma$ free field realization of $\widehat{sl}(n)$.

1. Introduction

Certain classes of low dimensional field theories are exactly soluble due to the presence of infinite dimensional Lie algebras in these models. Besides their appearance in 2-dimensional conformal field theories or their 3-dimensional topological counterparts, also the massive, respectively, non-topological perturbations thereof are expected to carry remnants of this algebraic structure. $W(g)$ algebras are, besides the affine Kac Moody algebras $\hat{g}$, the second known class of infinite dimensional Lie algebras descending from simple finite dimensional ones g [1–4]. In general they are intrinsically non-linear in that the commutation relations close only on the enveloping algebra of the modes of the generating fields. This accounts for both the variety of applications, as well as certain difficulties in handling them.

In particular, the construction of realizations in terms of an underlying linear oscillator or affine algebra is non-trivial. The major obstruction lies in proving that the algebra of the proposed field generators closes. Associativity is then guaranteed by the associativity of the underlying oscillator or affine algebra. The existence of a realization turns out to be closely related to the structure of a characteristic Hilbert space $\mathscr{H}(g)$ associated with it. The space $\mathscr{H}(g)$ encodes the information about the operator product expansion of the proposed set of generating fields. If $\mathscr{H}(g)$ contains a sufficient number of independent states (w.r.t. some graduation),