

On Certain Infinite Dimensional Aspects of Arakelov Intersection Theory

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Received June 14, 1991

Abstract. Let $k: Y \rightarrow X$ be an embedding of compact complex manifolds. Bismut and Lebeau have calculated the Quillen norm of the canonical isomorphism identifying the determinant of the cohomology of a holomorphic vector bundle over Y and the determinant of the cohomology of a resolution by a complex of holomorphic vector bundles over X . The purpose of this paper is to show that the formula of Bismut-Lebeau can be viewed as an equivariant intersection formula over the loop space of the considered manifolds, in the presence of an infinite dimensional excess normal bundle. This excess normal bundle is responsible for the appearance of the additive genus R of Gillet and Soulé in the formula of Bismut and Lebeau.

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