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Global Existence and Exponential Stability of Small Solutions to Nonlinear Viscoelasticity

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Abstract. The global existence of smooth solutions to the equations of nonlinear hyperbolic system of 2nd order with third order viscosity is shown for small and smooth initial data in a bounded domain of n-dimensional Euclidean space with smooth boundary. Dirichlet boundary condition is studied and the asymptotic behaviour of exponential decay type of solutions as t tending to ∞ is described. Time periodic solutions are also studied. As an application of our main theorem, nonlinear viscoelasticity, strongly damped nonlinear wave equation and acoustic wave equation in viscous conducting fluid are treated.

1. Introduction

In this paper, we are concerned with the global existence and exponential stability of small and smooth solutions to the following equations:

$$A_0(U)\partial_t^2 u + A_j(U)\partial_j\partial_t u - A_{ij}(U)\partial_i\partial_j u - B_{ij}(U)\partial_i\partial_j\partial_t u = f \qquad \text{in } [0, \infty)\times\Omega\,, \tag{1.1}$$

$$u = 0 \qquad \text{on } [0, \infty)\times\partial\Omega\,, \tag{1.2}$$

$$u(0, x) = u_0(x) \quad \text{and} \quad u_t(0, x) = u_1(x) \quad \text{in } \Omega\,. \tag{1.3}$$

The existence of time periodic solutions is also studied. Here, Ω is a bounded domain in $\mathbb{R}^n$ with C^∞ boundary $\partial\Omega$, $U = (\nabla u, u_t, \nabla u_t)$, $u_t = \partial_t u = \partial u/\partial t$, $\nabla u = (\partial_1 u, \dots, \partial_n u)$, $\partial_j u = \partial u/\partial x_j$ $(j = 1, \dots, n)$, $x = (x_1, \dots, x_n)\in\mathbb{R}^n$, $u = {}^t(u_1, \dots, u_d)$ is a d-vector of real-valued functions (${}^tM$ means the transposed M), and the summation convention is understood where the indices run through 1 to n. The $A_0(U)$, $A_j(U)$, $A_{ij}(U)$ and $B_{ij}(U)$ are $d\times d$ matrices of real-valued functions defined on $\{U\in\mathbb{R}^{(2n+1)d} \mid |U| \leqq K\}$ and in C^∞ there, which satisfies the following assumption: