Commun. Math. Phys. 148, 169–176 (1992)

Communications in
Mathematical
Physics

Topological Particle Field Theory, General Coordinate Invariance and Generalized Chern–Simons Actions

Noboru Kawamoto and Yoshiyuki Watabiki
Department of Physics, Kyoto University, Kyoto 606, Japan

Received October 14, 1991; in revised form January 10, 1992

Abstract. We show that recently proposed generalized Chern–Simons action can be identified with the field theory action of a topological point particle. We find the crucial correspondence which makes it possible to derive the field theory actions from a special version of the generalized Chern–Simons actions. We provide arguments that the general coordinate invariance in the target space and the flat connection condition as a topological field theory can be accommodated in a very natural way. We propose series of new gauge invariant observables.

Topological field theories so far proposed can be mainly classified into three classes: the standard Chern–Simons type, the action having a form of total derivative, and the vanishing action with a specific gauge fixing like, flat connection, self-duality condition, etc. [1, 2]. Here we propose a new type of topological field theory. We start from a vanishing particle theory action and impose a particular gauge fixing and consider the corresponding field theory action, which turns out to coincide with the generalized Chern–Simons actions derived in the previous paper [3] which we refer to as paper I. The general coordinate invariance and the topological nature of the actions are natural consequences of the formulations. There have been several investigations to point out a particular connection between the three-dimensional Chern–Simons action and a particle mechanics [4] and to clear up the various issues of the topological particle [5].

We first consider a point particle theory which is invariant under the following transformation:

$$x^\mu(\tau) \to x^\mu(\tau) + a^\mu(\tau) , \tag{1}$$

where $a^\mu(\tau)$ is an arbitrary function of τ which parametrizes the world line of a point particle moving in the N-dimensional target space. An obvious candidate of the action will be given by

$$S^{[1]} = 0 , \tag{2}$$

which is apparently invariant under the transformation (1) and thus has a vast corresponding symmetry. As a gauge fixing condition we take

$$\dot{x}^\mu(\tau) = 0 , \tag{3}$$