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Vector Fields on Complex Quantum Groups

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Abstract. Using previous results we construct the q-analogues of the left invariant vector fields of the quantum enveloping algebra corresponding to the complex Lie algebras of type A_{n-1}, B_n, C_n, and D_n. These quantum vector fields are functionals over the complex quantum group $\mathscr{A}$. In the special case A_1 it is shown that this Hopf algebra coincides with $U_q sl(2, \mathbb{C})$.

1. Introduction

We work with the q-deformed function algebras over the complexified groups associated to A_{n-1}, B_n, C_n, and D_n, where $q > 0$ is a real parameter. I.e. we consider Hopf algebras which are generated by the matrix functions of the fundamental representation and its hermitian conjugate such that dividing out the unitarity condition yields the quantum groups $SU_q(N)$, $SO_q(N, \mathbb{R})$, $USp_q(N)$. In [DSWZ] a dual Hopf algebra has been constructed thus leading to a q-deformation of the corresponding universal enveloping algebra. In [SWZ, OSWZ] the q-deformed universal enveloping algebra of $sl(2, \mathbb{C})$ was found as an operator algebra on the complex spinor quantum plane. This was also constructed in [CW] by analyzing the differential calculus on the complex quantum groups $Sl_q(n, \mathbb{C})$.

In the real case it is known that the Hopf algebra of regular functionals is generated in some sense by the vector fields which appear in the bicovariant differential calculus on quantum groups [Wor, Jur, Zum, CSWW]. This is proved in [Bur] using the fact that the matrices $L^+{}^i{}_j$ and $L^-{}^i{}_j$ generating the algebra of regular functionals are upper and lower triangular, respectively.

In the complex case the corresponding matrices $L^{\pm I}{}_J$ introduced in [DSWZ] violate this triangularity. In this paper we prove for the case of A_1 that the $*$-Hopf algebra of regular functionals is generated by the vector fields.

In Sect. 2 we define the vector fields, find some relations between them and construct the Casimir operators of the algebra of regular functionals $U_{\mathscr{R}}$ on the complex quantum group $\mathscr{A}$. In Sect. 3 we concentrate on the case A_1 and show