

# Non-Commutative Spheres

## III. Irrational Rotations

Ola Bratteli<sup>1,\*</sup> and Akitaka Kishimoto<sup>2</sup>

<sup>1</sup> Institute of Mathematics, University of Trondheim, N-7034 Trondheim – NTH, Norway

<sup>2</sup> Department of Mathematics, Hokkaido University, Sapporo, 060 Japan

Received October 3, 1991; in revised form December 11, 1991

*Dedicated to Professor Huzihiro Araki on the occasion of his 60<sup>th</sup> birthday*

**Abstract.** Let  $A_\theta$  be the irrational rotation algebra, i.e. the  $C^*$ -algebra generated by two unitaries  $U, V$  satisfying  $VU = e^{2\pi i\theta}UV$ , with  $\theta$  irrational, and consider the fixed point subalgebra  $B_\theta$  under the flip automorphism  $U \rightarrow U^{-1}, V \rightarrow V^{-1}$ . We prove that  $B_\theta$  is an AF-algebra.

### Table of Contents

1. Introduction . . . . .	605
2. Putnam’s Tower Construction on $\mathbb{T}$ . . . . .	607
3. A Subsidiary Tower Construction . . . . .	609
4. Kumjian’s Projections . . . . .	611
5. Finite-Dimensional Subalgebras . . . . .	613
6. Homogeneous Subalgebras . . . . .	614
7. Basic Building Blocks . . . . .	620
8. Small Eigenvalue Variation . . . . .	623
References . . . . .	624

### 1. Introduction

In this paper we continue the study, begun in [BEEK 1] and [BEEK 2], of the fixed point subalgebra of the rotation algebra under the flip. Recall from [Rie] that the rotation algebra  $A_\theta$  is the universal  $C^*$ -algebra generated by two unitaries  $U, V$  satisfying  $VU = \varrho UV$ , where  $\varrho = e^{2\pi i\theta}$  and  $0 \leq \theta < 1$ . The flip  $\sigma$  is the automorphism of this algebra defined through the requirements

$$\sigma(U) = U^{-1}, \quad \sigma(V) = V^{-1}. \tag{1.1}$$

\* Present address: Department of Mathematics, University of Oslo, P.O. Box 1053, N-0316 Oslo 3, Norway