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Chern-Simons-Witten Invariants of Lens Spaces and Torus Bundles, and the Semiclassical Approximation

Lisa C. Jeffrey[⋆]

Department of Pure Mathematics and Mathematical Statistics, University of Cambridge, 16 Mill Lane, Cambridge CB2 1SB, UK[⋆⋆]

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Abstract. We derive explicit formulas for the Chern-Simons-Witten invariants of lens spaces and torus bundles over S^1, for arbitrary values of the level k. Most of our results are for the group $G = SU(2)$, though some are for more general compact groups. We explicitly exhibit agreement of the limiting values of these formulas as $k \to \infty$ with the semiclassical approximation predicted by the Chern-Simons path integral.

1. Introduction

New invariants of 3-manifolds were introduced by Witten [40] using Chern-Simons gauge theory. In this paper we study these invariants explicitly for certain families of three-manifolds. Our ultimate objective is to verify for these families certain properties of the invariants derived heuristically using the Feynman path integral.

Witten [40] specified his three-manifold invariants in terms of the axioms of topological quantum field theory (TQFT), based on *modular functors* derived from conformal field theory [30]. This definition can be made rigorous, and is the one we shall use here. Three-manifold invariants were also defined combinatorially by Reshetikhin and Turaev [36], using modular Hopf algebras associated to quantum groups. Many properties of the invariants of Reshetikhin and Turaev have been established by Kirby and Melvin [23] from this point of view. It had always been expected that Reshetikhin and Turaev's definition would prove to be equivalent to the definition using the topological field theory axioms: this was, however, only proved recently by Walker ([39], Sects. 9–11).

Pure mathematicians who have worked on these invariants have focused almost exclusively on the combinatorial or TQFT definition. On the other hand, Witten and

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⋆⋆ Address as of September 1, 1991: School of Natural Science, Institute for Advanced Study, Princeton, NJ 08540; USA