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The Isomonodromy Approach to Matrix Models in 2D Quantum Gravity

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Abstract. We consider the double-scaling limit in the hermitian matrix model for 2D quantum gravity associated with the measure $\exp \sum_{j=1}^{N} t_j z^{2j}$, $N \geqq 3$. We show that after the appropriate modification of the contour of integration the Cross-Migdal-Douglas-Shenker limit to the Painlevé I equation (in the generic case of the pure gravity) is valid and calculate the nonperturbative parameters of the corresponding Painlevé function. Our approach is based on the WKB-analysis of the L-A pair corresponding to the discrete string equation in the framework of the Inverse Monodromy Method. Here we extend our results, which were obtained before for the particular cases $N=2,3$. Our analysis complements the isomonodromy approach proposed by G. Moore to the general string equations that come from the matrix model in the continuous limit and differ in that we apply the isomonodromy technique to investigate the double scaling limit itself.

1. Introduction

We shall study the difference equation

$$n = w_n^{1/2} \sum_{j=1}^{N} j t_j (L^{2j-1})_{n,n-1}, \qquad L_{nm} \doteqdot \frac{1}{2} w_m^{1/2} \delta_{n+1,m} + \frac{1}{2} w_n^{1/2} \delta_{n-1,m}, \tag{1.1}$$

where $n \in \mathbb{Z}$, $t_j \in \mathbb{C}$, $1 \leqq j \leqq N$, $N \geqq 3$, are regarded given parameters, and L is the operator acting in the space $\psi = \{\psi_n\}_{n=-\infty}^{\infty}$ via $(L\psi)_n = \sum_{m=-\infty}^{\infty} L_{nm}\psi_m$. This nonlinear equation for the dependent variable $w_n \in \mathbb{C}$ has recently appeared in connection with a matrix model in 2D quantum gravity [1, 2] and for this reason we shall refer to it as the **discrete string equation**. We will outline, following [3], the

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