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Localization in the Ground State of a Disordered Array of Quantum Rotators

Abel Klein[1] **and J. Fernando Perez**[2]
Department of Mathematics, University of California at Irvine, Irvine, CA 92717, USA

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Abstract. We consider the zero-temperature behavior of a disordered array of quantum rotators given by the finite-volume Hamiltonian:

$$H_\Lambda = -\sum_{x\in\Lambda}\frac{h(x)}{2}\frac{\partial^2}{\partial\varphi(x)^2} - J\sum_{\langle x,y\rangle\in\Lambda}\cos(\varphi(x)-\varphi(y))\,,$$

where $x, y\in\mathbf{Z}^d$, $\langle\, ,\rangle$ denotes nearest neighbors in $\mathbf{Z}^d$; $J>0$ and $\mathbf{h}=\{h(x)>0, x\in\mathbf{Z}^d\}$ are independent identically distributed random variables with common distribution $d\mu(h)$, satisfying $\int h^{-\delta}d\mu(h)<\infty$ for some $\delta>0$. We prove that for any $m>0$ it is possible to choose $J(m)$ sufficiently small such that, if $0<J<J(m)$, for almost every choice of $\mathbf{h}$ and every $x\in\mathbf{Z}^d$ the ground state correlation function satisfies

$$\langle\cos(\varphi(x)-\varphi(y))\rangle \leqq C_{x,\mathbf{h},J}\,e^{-m|x-y|}$$

for all $y\in\mathbf{Z}^d$ with $C_{x,\mathbf{h},J}<\infty$.

1. Introduction

Ferromagnetically coupled quantum rotators have been used in the physics literature to describe the effect of quantum fluctuations in granular superconductors [1]. In this paper we discuss the typical properties of a disordered array of such rotators with random moments of inertia. Apart from its intrinsic physical interest the study of this model is a natural step in the program initiated in [2] and [3] of understanding the effect of randomness in quantum spin systems. In [2], Klein and Perez studied the ground state of the one-dimensional quantum x–y model in the presence of a random transverse field: exponential decay of the correlation

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[2] Permanent Address: Instituto de Fisica, Universidade de Sao Paulo, P.O. Box 20516, 01498 Sao Paulo, Brazil. Partially Supported by CNPq under grant 303795-77FA