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Instability for the Navier–Stokes Equations on the 2-Dimensional Torus and a Lower Bound for the Hausdorff Dimension of Their Global Attractors

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Abstract. We prove instability of stationary solutions of the Navier–Stokes equations on the domain $[0, 2\pi] \times [0, 2\pi]$ with periodic boundary condition for a class of external forces for large Reynolds number. Moreover, we give a lower bound for the Hausdorff dimension of the global attractors.

1. Introduction

This paper is a continuation of our previous work [9]. We consider the two dimensional Navier–Stokes equations for a viscous incompressible fluid with spatially periodic boundary conditions (with periods $2\pi, 2\pi$). The Navier–Stokes equations with velocity u and external force f (assume f is time independent) in functional form can be written as (see [3, 14, 15])

$$\frac{du}{dt} + Au + B(u, u) = f, \tag{1}$$

$$u(0) = u_0, \tag{2}$$

in a Hilbert space H, where H consists of those u such that

$$u = \sum_{j=(j_1, j_2) \in Z^2} u_j e^{i(j_1 x_1 + j_2 x_2)}, \quad u_j \in C^2, \quad u_{-j} = \bar{u}_j, \quad u_0 = 0, \tag{3}$$

$$j * u_j = 0, \quad \text{for each } j, \tag{4}$$

$$|u|^2 = (2\pi)^2 \sum_{j \in Z^2} |u_j|^2 < \infty. \tag{5}$$

Let P be the orthogonal projection onto H in $(L^2(\Omega))^2$ (where $\Omega = [0, 2\pi] \times [0, 2\pi]$), then

$$Au = -P\Delta u,$$

$$B(v, w) = P[(v * \nabla)w].$$