Commun. Math. Phys. 147, 211–216 (1992)

The Energy Operator for Infinite Statistics

Sonia Stanciu★

Physikalisches Institut der Universität Bonn, Nußallee 12, W-5300 Bonn 1, FRG

Received March 19, 1992

Abstract. We construct the energy operator for particles obeying infinite statistics defined by a q-deformation of the Heisenberg algebra.

The aim of this paper is to construct the energy operator for particles which obey the so-called infinite statistics defined by the q-deformation of the Heisenberg algebra. This topic was studied in the previous article [1], where a conjecture was formulated concerning the form of the energy operator. Our main result is a proof of this conjecture in a slightly modified form (cf. Remark 1).

We will essentially use the same notations as in [1]. Thus, T_{1k} will denote the particular elements of $\mathfrak{S}_n$ which send $[1, 2, \ldots, n]$ to $[k, 1, \ldots, k-1, k+1, \ldots, n]$, i.e.

$$T_{1k}(i) = \begin{cases} k, & \text{if } i = 1; \\ i-1, & \text{if } 1 < i \leq k; \\ i, & \text{if } k < i \leq n; \end{cases}$$

and $\mathfrak{S}_{n,p}$ will represent the following subsets of $\mathfrak{S}_n$:

$$\mathfrak{S}_{n,p} = \{\sigma \in \mathfrak{S}_n, \ \text{ with } \sigma = T_{1k_1} T_{1k_2}, \ldots, T_{1k_p}, \ 1 < k_1 < \ldots < k_p \leq n\},$$

for $1 \leq p \leq n-1$ and $\mathfrak{S}_{n,0} = \{1\}$. (This differs from the definition of $\mathfrak{S}_{n,p}$ in [1].)

In [1] an $n! \times n!$ matrix $A_n(\pi, \sigma)$, π, $\sigma \in \mathfrak{S}_n$, with coefficients in $\mathbb{Z}[q]$ was studied and shown to be invertible for $|q| < 1$. As in [1], we will work with the group algebra $\mathbb{C}[\mathfrak{S}_n]$ rather than its matrix representation, so we have elements

$$\alpha_n = \sum_{\varrho \in \mathfrak{S}_n} A_n(\varrho, 1)\sigma = \sum_{\varrho \in \mathfrak{S}_n} q^{I(\varrho)} \varrho, \qquad \alpha_n^{-1} = \sum_{\varrho \in \mathfrak{S}_n} A_n^{-1}(\varrho, 1)\,\varrho\,. \tag{1}$$

Let $\mathscr{E}$ be the energy operator of particles obeying infinite statistics, defined by the commutation relation (1) in [1]. $\mathscr{E}$ acts on $\mathscr{H}(q)$ and each x_1 is an eigenvector of $\mathscr{E}$

★ e-mail: stanciu@pib1.physik.uni-bonn.de