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The Semiclassical Limit for Gauge Theory on S^2 ★

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Abstract. It is shown that the Yang-Mills measure $Z_h^{-1}e^{-S(\omega)/h}[D\omega]$, where $h>0$, describing gauge fields on the two-sphere converges to a probability measure on the moduli space of Yang-Mills connections on S^2, as $h\to 0$.

1. Introduction

In this paper we prove that the quantum Yang-Mills measure $d\mu_{\mathrm{YM}}^T(\omega) = \frac{1}{Z_T}e^{-S(\omega)/T}[D\omega]$ (notation to be explained in Sect. 2) for gauge fields over the two-sphere S^2 converges, as $T\to 0$, to a probability measure μ_{YM}^T on the set of minima of the Yang-Mills action functional S. The measure μ_{YM}^T has been constructed and studied in [Se 1, 2] (and, from a different point of view, by Fine in [F]) for a wide class of gauge groups. On the other hand, the minima of the Yang-Mills action S for gauge fields over S^2 are also well-understood [AB, G, FH, Se 1, NU]. In Sect. 2 we summarize the relevant results that are known and in Sect. 3 we describe the limiting process.

2. Classical and Quantum Yang-Mills on S^2

Let G be a compact connected Lie group with a fixed bi-invariant metric $\langle \cdot, \cdot \rangle_{\mathfrak{g}}$ on its Lie algebra $\mathfrak{g}$.

Equip S^2 with a Riemannian metric. If E is a Borel subset of S^2 we denote by $|E|$ its area as given by the area-measure $d\sigma$ induced by the metric. For the geometric discussions we will visualize S^2 as the usual sphere sitting in R^3 and we will equip it with a north pole n, a south pole s, and the hemispheres N and S which intersect in the equator $\mathscr{E}$. We will often refer to the meridians – these are the usual meridians

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