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Self-Avoiding Walk in Five or More Dimensions I. The Critical Behaviour

Takashi Hara[1]* **and Gordon Slade**[2]**

[1] Department of Physics, Gakushuin University, Toshima-ku, Tokyo 171, Japan
[2] Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario, Canada L8S 4K1

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Abstract. We use the lace expansion to study the standard self-avoiding walk in the d-dimensional hypercubic lattice, for $d \geqq 5$. We prove that the number c_n of n-step self-avoiding walks satisfies $c_n \sim A\mu^n$, where μ is the connective constant (i.e. $\gamma = 1$), and that the mean square displacement is asymptotically linear in the number of steps (i.e. $\nu = 1/2$). A bound is obtained for $c_n(x)$, the number of n-step self-avoiding walks ending at x. The correlation length is shown to diverge asymptotically like $(\mu^{-1} - z)^{-1/2}$. The critical two-point function is shown to decay at least as fast as $|x|^{-2}$, and its Fourier transform is shown to be asymptotic to a multiple of k^{-2} as $k \to 0$ (i.e. $\eta = 0$). We also prove that the scaling limit is Gaussian, in the sense of convergence in distribution to Brownian motion. The infinite self-avoiding walk is constructed. In this paper we prove these results assuming convergence of the lace expansion. The convergence of the lace expansion is proved in a companion paper.

Contents

* Supported by the Nishina Memorial Foundation and NSF grant PHY-8896163. Present address: Department of Applied Physics, Tokyo Institute of Technology, Oh-Okayama, Meguro-ku, Tokyo 152, Japan, electronic address: hara@appana.ap.titech.ac.jp
** Supported by NSERC grant A9351, electronic address: slade@aurora.math.mcmaster.ca