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A Criterion for Flatness in Minimal Area Metrics that Define String Diagrams

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Abstract. It has been proposed that the string diagrams of closed string field theory be defined by a minimal area problem that requires that all nontrivial homotopy curves have length greater than or equal to 2π. Consistency requires that the minimal area metric be flat in a neighbourhood of the punctures. The theorem proven in this paper, yields a criterion which if satisfied, will ensure this requirement. The theorem states roughly that the metric is flat in an open set, U if there is a unique closed curve of length 2π through every point in U and all of these closed curves are in the same free homotopy class.

1. Introduction and Motivation

Consistency Conditions on String Field Theory. The fundamental idea of string theory is to define an n string amplitude as an integral over the moduli spaces of n punctured Riemann surfaces of all genera. For on-shell states the integrand arises from a conformal field theory. In fact the integrand is just the correlation function of n vertex operators inserted at the location of the punctures. The operator formulation of conformal field theory, which defines correlation functions as vacuum expectation values of a string of quantum fields, does not tell us how to extend the definition of amplitudes to off-shell states. This is done naturally in the path integral approach where we have a prescription for calculating general amplitudes (off and on shell). Fundamental to this prescription is the notion of a string diagram. A string diagram is a Riemann surface together with a choice of local coordinate (up to phase ambiguity) around each puncture. In fact for the purposes of this paper it is sufficient to say that the problem of defining off-shell string amplitudes is one of defining string diagrams [SoZ]. It is necessary that the local coordinates be ambiguous up to phase if the string diagrams are to be globally well defined on moduli space. We want amplitudes that are independent of the phase that we choose around each

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