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Decay of Correlations for Certain Quadratic Maps*

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Abstract. We prove exponential decay of correlations for (f, μ), where f belongs in a positive measure set of quadratic maps of the interval and μ is its absolutely continuous invariant measure. These results generalize to other interval maps.

Consider a dynamical system generated by a map $f: M \to M$ preserving a probability measure μ, and let $\varphi, \psi: M \to \mathbb{R}$ be observables. Mixing properties of the dynamical system are reflected in the decay of correlations between φ and $\psi \circ f^n$ as $n \to \infty$. More precisely, we say that (f, μ) has *exponential decay of correlations* for functions belonging in a certain class X if there is a number $\tau < 1$ such that for every $\varphi, \psi \in X$, there is a constant $C = C(\varphi, \psi)$ such that

$$\left|\int \varphi \cdot (\psi \circ f^n) d\mu - \int \varphi d\mu \cdot \int \psi d\mu\right| \leqq C\tau^n \quad \forall n \geqq 0.$$

The main result of this paper is the following:

Theorem. *Consider $f_a: [-1, 1] \circlearrowleft$ defined by $f_a(x) = 1 - ax^2, a \in [0, 2]$. Then there is a positive Lebesgue measure set Δ in parameter space such that if $f = f_a$ for $a \in \Delta$, then*

(1) *f has an absolutely continuous invariant measure μ (this is a well known theorem first proved by Yakobson* [J]);

(2) *(f, μ) has exponential decay of correlations for functions of bounded variation*;

(3) *the central limit theorem holds for $\{\varphi \circ f^n\}_{n=1,2,\ldots}, \varphi \in BV$.*

These results generalize to certain open sets of 1-parameter families of unimodal maps.

Exponential decay of correlations has been proved for primarily two types of dynamical systems: piecewise uniformly expanding maps of the interval with their absolutely continuous invariant measures, and Axiom A diffeomorphisms with their Gibbs states. (See e.g. [HK, Ry1, Ru1, Ru2].) These are by no means the only results. (See e.g. [BS], [Z].)

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