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Quantum Riemann Surfaces I. The Unit Disc*

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Abstract. We construct a non-commutative $\mathbb{C}^*$-algebra $C_\mu(\bar{U})$ which is a quantum deformation of the algebra of continuous functions on the closed unit disc $\bar{U}$. $C_\mu(\bar{U})$ is generated by the Toeplitz operators on a suitable Hilbert space of holomorphic functions on U.

1. Introduction

Alain Connes has shown [8] that a substantial part of differential geometry can be extended to a non-commutative setup in which a non-commutative $*$-algebra replaces an algebra of functions on a manifold. Clearly, not every non-commutative algebra has an interesting geometry and, while a satisfactory concept of a "non-commutative differentiable manifold" has not been formulated yet, it is desirable to study examples of such structures.

Recently, a growing number of examples of "non-commutative differentiable manifolds" has been studied, see e.g. [16, 18, 22]. These examples form, in a sense, a testing ground for probing general concepts of non-commutative differential geometry and so give us a better insight into the properties of a non-commutative manifold. Also, some important applications of these ideas have been found in other areas of mathematics and physics [3, 7, 9].

In this paper, we begin a program of developing a theory of non-commutative Riemann surfaces. We believe that non-commutative Riemann surfaces should play a distinguished role in non-commutative differential geometry, very much like ordinary Riemann surfaces in the commutative case. Also, there are some speculations [13] that quantum Riemann surfaces might be helpful in studying certain integrable systems arising in string theory.

Conceptually, the simplest method of constructing non-commutative manifolds is the framework of deformation quantization, see e.g. [2, 4, 19, 22]. This is the starting point of our approach.

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