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# $\bar{\partial}$-Torsion for Complex Manifolds and the Adiabatic Limit

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**Abstract.** We consider a complex fibration $F \to M \xrightarrow{\pi} B$ and pull back bundles $E_1$ and $E_2$ over $M$. Using the adiabatic limit idea, we compute the metric invariant $T_p(E_1)/T_p(E_2)$, where $T_p(E)$ denotes the complex Ray–Singer torsion.

## 1. Introduction

In this paper we will study the Ray–Singer $\bar{\partial}$-torsion [RSI&II] of complex manifolds which are fiber bundles. To be more specific, we consider the fibration

$$\begin{array}{c} F \to M \\ \quad \downarrow \pi, \\ \quad B \end{array} \tag{1.1}$$

where $F, B$ and $M$ are compact complex manifolds and where $\pi$ is holomorphically locally trivial. Given a hermitian bundle $\mathscr{E}$ over $B$, one wants to compute the $\bar{\partial}$-torsion $T_p(\pi^*\mathscr{E})$ on $M$. We recall that the torsion is defined as follows:

$$T_p(\pi^*\mathscr{E}) = \exp\left(\frac{d}{ds}\bigg|_{s=0} \sum (-1)^q q \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1} tr e^{-t\Delta^{p,q}} dt\right), \tag{1.2}$$

where $\Delta^{p,q}$ is the $\bar{\partial}$-Laplacian on forms in $\Lambda^{p,q}T^*M \otimes \pi^*\mathscr{E}$ and where the right-hand side of (1.2) is to be understood as the derivative at $s = 0$ of the analytic continuation of the sum. (For more details, we refer to [RSI] and Seeley [Se].) We will use the convention that $\Delta$ is a positive operator. Note also that (1.2) defines the $\bar{\partial}$-torsion to be the square of that of [RSI&II]. In the case where the bundles are constructed by representations of $\pi_1(M)$, J. Fay [Fa], showed that (1.2) allowed to define analytic extensions of the torsion to non unitary representations.

One wishes to find formulas for the torsion in terms of the torsion of the base manifold. In the real case, such formulas are known. They can either be proved topologically (see D. Fried [Fr]) or analytically (see Forman [Fo] or Dai, Epstein and Melrose [DEM]). For acyclic pull-back bundles, these formulas are remark-