Commun. Math. Phys. 145, 605–634 (1992)

Communications in Mathematical Physics



# A Characterization of "Rapidity" Curve in the Chiral Potts Model

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Received August 12, 1991

**Abstract.** We study the geometry of high genus curves of rapidity variables in chiral Potts model. In terms of symmetries, we characterize these Rieman surfaces and derive their period matrices. By the theory of prime forms, the temperature-like parameter is expressed by hyperelliptic theta functions.

## Introduction

Nowadays much attention is paid to the exactly solvable models of statistical mechanics. The integrable $N$-state chiral Potts model is the first solvable model where the Boltzmann weights lie on a complex curve of genus greater than one. It contains the natural generalization of Ising model and many of the remarkable properties known for the Ising model ($N=2$) can be extended to the cases $N \geqq 3$ (see refs. [1, 5, 8, 15] and references therein). This model gives the solution of Yang-Baxter equation (or the star-triangle equation) which has a very simple product form for the Boltzmann weights

$$\frac{W_{pq}(n)}{W_{pq}(0)} = \prod_{j=1}^{n} \frac{d_p b_q - a_p c_q \omega^j}{b_p d_q - c_p a_q \omega^j},$$

$$\frac{\overline{W_{pq}}(n)}{\overline{W_{pq}}(0)} = \prod_{j=1}^{n} \frac{\omega a_p d_q - d_p a_q \omega^j}{c_p b_q - b_p c_q \omega^j},$$

depending on the two "rapidities" $p$, $q$ and a parameter $k'$, where

$$\omega = e^{\frac{2\pi i}{N}}, \qquad j\text{: integer mod. } N.$$

The "rapidity" variables $p$, $q$ are represented by the ratio of 4-vectors $[a, b, c, d]$ satisfying

$$ka^N + k'c^N = d^N,$$

$$kb^N + k'd^N = c^N,$$