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BRST Analysis of Physical States for 2D Gravity Coupled to $c \leqq 1$ Matter

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Abstract. We consider 2D gravity coupled to $c \leqq 1$ conformal matter in the conformal gauge. The Liouville system is represented by a free scalar field, ϕ^L, with background charge such that the BRST operator imposing reparametrization invariance is nilpotent. We compute the cohomology of this BRST charge on the product of the Fock space of ϕ^L with those of the ghosts and one other free scalar field, ϕ^M, representing the matter system. From this calculation the physical states of the full theory are determined. For the $c < 1$ case the further projection from the Fock space of ϕ^M to the irreducible representation, using Felder's resolution, reproduces the results of Lian and Zuckerman.

1. Introduction

Matrix model techniques appear to give a great deal of information on discretized gravity coupled to $c^M \leqq 1$ matter in two dimensions. To put these insights to use it is clearly necessary to understand the continuum theory, and thus the Liouville dynamics to which it reduces in the chiral gauge [1, 2], and conformal gauge [3, 4]. Upon gauge fixing, the continuum theory factorizes into matter, Liouville, and ghost sectors coupled by the BRST constraint imposing diffeomorphism invariance. Indeed in the same way the relevance of Liouville dynamics to string theory was already well understood [5], and several groups went on to study the Liouville theory in detail [6–9]. From the work in [8–10], and as is consistent with semiclassical calculations [11–14], free field techniques may be used to advantage in the Liouville theory. Further, the free field description of minimal matter theories is well developed [15–17]. Application of these techniques to the full theory requires at least the description of the physical spectrum, and construction of the correlators of physical operators (see e.g. [18–25] for some recent developments). In this paper we discuss the former.

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