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Lyapunov Exponents of the Schrödinger Equation with Quasi-Periodic Potential on a Strip

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Abstract. We prove that all the non-negative Lyapunov exponents of difference Schrödinger equation

$$-y_{n+1}+Q_n y_n - y_{n-1}=0, \qquad -\infty<n<+\infty$$

are strictly positive. Here $y_n \in R^m$ and Q_n is a symmetric $m\times m$ matrix whose off-diagonal elements do not depend on n, and the diagonal elements are quasi-periodic functions

$$q_{nj}(\theta)=\lambda f_j(e^{2\pi i(\theta+n\alpha)})-E$$

with all f_i non-constant analytic functions, λ sufficiently large, and α any irrational number.

1. Introduction and Formulation of Results

In this paper we shall study the Lyapunov exponents of the difference equation:

$$-y_{n+1}+Q_n y_n - y_{n-1}=0, \qquad -\infty<n<+\infty, \tag{1}$$

where $y_n \in R^m$ and Q_n is a symmetric $m\times m$ matrix whose off-diagonal elements do not depend on n, and the diagonal elements are quasi-periodic functions

$$q_{nj}(\theta)=\lambda f_j(e^{2\pi i(\theta+n\alpha)})-E$$

with $f_j(z)$ non-constant analytic on $\mathscr{A}\equiv\{z\,|\,r<|z|<1/r\}$, taking values in $[-1,1]$ for $|z|=1$, λ is a (large) parameter called coupling constant, E is the energy, and α is any irrational number. Without loss of generality we shall assume that $\max_{1\leq i\leq m}\ \sup_{|z|=1} f_i(z)=1$ and $\min_{1\leq i\leq m}\ \inf_{|z|=1} f_i(z)=-1$.

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