Commun. Math. Phys. 145, 329–344 (1992)

Communications in Mathematical Physics

String Vertices, Overlap Equations, τ Functions and the Hirota Equation

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Received July 2, 1991; in revised form August 23, 1991

Abstract. String vertices, V, are shown to satisfy a new type of overlap equation of the form $V\exp\{ip\cdot Q^i(\xi^i)\} = V\exp\{ip\cdot Q^j(\xi^j)\}\left(\frac{d\xi^j}{d\xi^i}\right)^{p^2/2}$ as well as corresponding equations for A_n and B_n cycles. A special case of such an equation, when integrated, is shown to be the Hirota equation for the K–P hierarchy.

1. Introduction

There are a number of different approaches to string theory; the dual model [1], the light-cone string field theory [2], the sum over world sheet surfaces (the Polyakov approach) [3], gauge covariant string field theory [4] and the new oscillator formalism. The latter approach was developed with the objectives of providing an efficient method of calculating string perturbation theory and giving in some sense, a more fundamental definition of string theory. While the extent to which the latter aim has been achieved is not clear, it did succeed in giving perhaps the most efficient method of computing string perturbation theory. The formalism has a number of features in common with the original dual model approach to string theory; in particular it works with multi-string vertices. However, in the new oscillator formalism ghost oscillators were introduced into the vertices [5]. Although four distinct groups worked on different variants of the new oscillator formalism, substantial use was made of a new kind of relations called *overlap equations* which were discovered in Refs. [5–7]. These equations came in two types called unintegrated overlap equations and integrated overlap equations. The integrated overlap equations were often subsequently called conserved charges in the literature.

One of the new oscillator formalisms was called the group theoretic approach [7, 8] since calculations were reduced to essentially an exercise in manipulating conformal transformations. The basis of this approach were the overlap equations and the decoupling of zero norm physical states. The relations between the different