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Finite Dimensional Representations of the Quantum Lorentz Group

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Abstract. All finite dimensional irreducible representations of the quantum Lorentz group $\mathrm{SL}_q(2,\mathbb{C})$ are described explicitly and it is proved all finite dimensional representations of $\mathrm{SL}_q(2,\mathbb{C})$ are completely reducible. The conjecture of Podleś and Woronowicz will be answered affirmatively.

0. Introduction

The quantum Lorentz group $\mathrm{SL}_q(2,\mathbb{C})$, where q is a real parameter $\neq 0, \pm 1$, was introduced by Podleś and Woronowicz [PW], and the Iwasawa decomposition and representation theory were studied. This quantum group is combined with the double group of $\mathrm{SU}_q(2)$, a q-analogue of the compact group SU(2) [W, MMNNU], through the Iwasawa decomposition. Let A_q (respectively B_q) be the *-Hopf algebra corresponding to the quantum group $\mathrm{SL}_q(2,\mathbb{C})$ (respectively $\mathrm{SU}_q(2)$). (A *-Hopf algebra means a Hopf algebra over $\mathbb{C}$ with a *-operation satisfying some properties. See Sect. 4.) The dual vector space $B'_q = \mathrm{Hom}_{\mathbb{C}}(B_q, \mathbb{C})$ has a topological Hopf algebra structure. By a *topological Hopf algebra*, we mean a topological analogue of the usual Hopf algebra, in which the underlying vector space is assumed to have a linear topology and the complete tensor product $\hat{\otimes}$ plays the role of the usual tensor product. (See Sect. 1.)

Podleś and Woronowicz have introduced some topological Hopf algebra structure as well as some *-operation on $B_q \hat{\otimes} B'_q$ and have proved there is an injective *-Hopf algebra map of A_q into $B_q \hat{\otimes} B'_q$. We call

$$\mathscr{E}_q = B_q \hat{\otimes} B'_q$$

the *quantum double* of B_q. This is the dual version of Drinfeld's quantum double [D], and corresponds to the double group of $\mathrm{SU}_q(2)$.

The topological Hopf algebra $\mathscr{E}_q$ has the largest (non-topological) Hopf subalgebra E_q, and what they have done is the construction of an injective *-Hopf algebra map of A_q into E_q. This is not surjective. There is a central group-like