

Expanding Direction of the Period Doubling Operator

Yunping Jiang¹, Takehiko Morita² and Dennis Sullivan³

¹ The Institute for Mathematical Sciences, SUNY at Stony Brook, Stony Brook, 11794, USA

² Department of Mathematics, Tokyo Institute of Technology, Ohokayama, Meguro, Tokyo 152, Japan

³ Department of Mathematics, Graduate Center of CUNY, 33 West 42nd Street, New York, NY 10036, USA

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Abstract. We prove that the period doubling operator has an expanding direction at the fixed point. We use the induced operator, a “Perron–Frobenius type operator,” to study the linearization of the period doubling operator at its fixed point. We then use a sequence of linear operators with finite ranks to study this induced operator. The proof is constructive. One can calculate the expanding direction and the rate of expansion of the period doubling operator at the fixed point.

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1. Introduction

Perron–Frobenius Operator. Suppose M^n is a compact, connected, oriented and smooth Riemannian manifold, $\Omega \subset M^n$ is an open set and $\sigma: \Omega \rightarrow M^n$ is an expanding mapping. Let $\mathcal{B} = \{v | v \text{ is a complex Lipschitz vector field on } \Omega\}$ and let φ be a real Lipschitz function on Ω . The Perron–Frobenius operator $\mathcal{L}_\varphi: \mathcal{B} \rightarrow \mathcal{B}$