

Topology, Unitary Representations and Charged Particles

George Papadopoulos^{*,**}

Mathematics Department, King's College London, London WC2R 2LS, UK

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Abstract. The quantum mechanics of the charged particles with rigid and local symmetries propagating on the manifold M is studied. It is shown that the classical rigid symmetries of this model may be anomalous. These anomalies are of local and global type, and they related to topological obstructions to lifting a group action of a group G on M to a principal $U(1)$ bundle P over M . The charged particles with local symmetries may have additional anomalies and the representation theory of the group G is used to study these anomalies. Finally, the quantum mechanics of the supersymmetric charged particles with symmetries is examined.

1. Introduction

The classical and quantum theory of a charged particle coupled to a $U(1)$ gauge field (magnetic field) has attracted much attention. This is due to the existence of quantum mechanical properties of this model which have topological interpretation.

There are two such quantum properties. The first one is Dirac's observation that the coupling constant of gauge field must be quantised [1]. The second property is the observation of the authors in refs. [2–4] that the symmetries of the classical equations of motion of this theory might be anomalous. In ref. [2], N. S. Manton observed that a $U(1)$ symmetry of a charged particle propagating on a flat torus is anomalous. In ref. [3], R. S. Ward extended this result to the $U(1)$ symmetries of a charged particle propagating on any manifold M . The anomaly was contributed to the fact that the $U(1)$ charges at the quantum level are locally defined operators. These anomalies were also observed in ref. [10] using path integral methods. Finally in ref. [4], R. Jackiw suggested that the algebra of

* e. mail address: udah130@oak.cc.kcl.ac.uk

** Address after September 1990: Physics Department, Queen Mary and Westfield College, Mile End, London E1 4NS