Commun. Math. Phys. 144, 443–490 (1992)

Communications in Mathematical Physics

Finitely Correlated States on Quantum Spin Chains

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Received July 12, 1990; in revised form June 3, 1991

Abstract. We study a construction that yields a class of translation invariant states on quantum spin chains, characterized by the property that the correlations across any bond can be modeled on a finite-dimensional vector space. These states can be considered as generalized valence bond states, and they are dense in the set of all translation invariant states. We develop a complete theory of the ergodic decomposition of such states, including the decomposition into periodic "Néel ordered" states. The ergodic components have exponential decay of correlations. All states considered can be obtained as "local functions" of states of a special kind, so-called "purely generated states," which are shown to be ground states for suitably chosen finite range VBS interactions. We show that all these generalized VBS models have a spectral gap. Our theory does not require symmetry of the state with respect to a local gauge group. In particular we illustrate our results with a one-parameter family of examples which are not isotropic except for one special case. This isotropic model coincides with the one-dimensional antiferromagnet, recently studied by Affleck, Kennedy, Lieb, and Tasaki.

1. Introduction

Determining ground state properties of quantum spin systems on a lattice is often a hard problem, and is certainly much more complex than the corresponding problem in classical statistical mechanics. One reason for this difference is that in a classical theory the energy of a state can be minimized locally, by fixing the state on the boundary $\partial\Lambda$ of a finite region Λ, and finding the local state in Λ of minimal energy with the prescribed marginals on the sites in $\partial\Lambda$. This procedure breaks down in a quantum system, because the local state obtained in this way, and the state outside Λ may fail to have a common extension [65]. A closely related point