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Holomorphy of the Scattering Matrix with Respect to c^{-2} for Dirac Operators and an Explicit Treatment of Relativistic Corrections

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Abstract. We prove holomorphy of the scattering matrix at fixed energy with respect to c^{-2} for abstract Dirac operators. Relativistic corrections of order c^{-2} to the nonrelativistic limit scattering matrix (associated with an abstract Pauli Hamiltonian) are explicitly determined. As applications of our abstract approach we discuss concrete realizations of the Dirac operator in one and three dimensions and explicitly compute relativistic corrections of order c^{-2} of the reflection and transmission coefficients in one dimension and of the scattering matrix in three dimensions. Moreover, we give a comparison between our approach and the first-order relativistic corrections according to Foldy-Wouthuysen scattering theory and show complete agreement of the two methods.

1. Introduction

We provide a general framework for the nonrelativistic limit of scattering theory for general Dirac operators. Our treatment is based on an abstract approach employed in [10, 11] to obtain explicit expressions for first order corrections of bound state energies with respect to c^{-2}.

Historically, the first rigorous treatment of the nonrelativistic limit of Dirac Hamiltonians seems to go back Titchmarsh [36] who proved holomorphy of the Dirac eigenvalues (rest energy subtracted) with respect to c^{-2} for spherically symmetric potentials and obtained explicit formulas for relativistic bound state corrections of order $O(c^{-2})$ (formally derived in [32]). Holomorphy of the Dirac resolvent in three dimensions in c^{-1} for electrostatic interactions were first obtained by Veselic [38] and then extended to electromagnetic interactions by Hunziker [16]. An entirely different approach, based on an abstract set up, has been used in [6] to prove strong convergence of the unitary groups as $c^{-1} \to \infty$. Employing this abstract framework, holomorphy of the Dirac resolvent in c^{-1} under general conditions on the electromagnetic interaction potentials has been obtained in [10, 11]. Moreover, this approach led to the first rigorous derivation of explicit formulas for relativistic corrections of order $O(c^{-2})$ to bound state