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On the Uniqueness of Static Perfect-Fluid Solutions in General Relativity*

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Abstract. Following earlier work of Masood-ul-Alam, we consider a uniqueness problem for non-rotating stellar models. Given a static, asymptotically flat perfect-fluid spacetime with barotropic equation of state $\varrho(p)$, and given another such spacetime which is spherically symmetric and has the same $\varrho(p)$ and the same surface potential: we prove that both are identical provided $\varrho(p)$ satisfies a certain differential inequality. This inequality is more natural and less restrictive than the conditions required by Masood-ul-Alam.

1. Introduction

In this paper we solve a uniqueness problem for non-rotating stellar models. Namely, given a static, asymptotically flat perfect-fluid spacetime with barotropic equation of state $\varrho(p)$ subject to certain restrictions: we show that this is unique provided there exists another such model [called "spherical reference (SR-) model"] which is spherically symmetric and has the same $\varrho(p)$ and the same value V_s of the gravitational potential on the boundary of the star ("surface redshift"). This problem was first raised and solved by Masood-ul-Alam [1] under more restrictive and somewhat unnatural conditions on $\varrho(p)$.

The main motivation for the present problem comes from the "fluid ball conjecture" which states that a non-rotating stellar model is spherically symmetric (see Künzle and Savage [2]). If this was known, it would follow that such a model consists of a family of solutions determined by one parameter, e.g. the value of the pressure at the centre. (For a rigorous proof of this fact see Schmidt [3].) In the absence of a direct proof of spherical symmetry for general $\varrho(p)$, one tries to turn this reasoning around by concluding sphericity from uniqueness for some given parameter, and here the surface redshift turns out to be a possible choice. However this parametrization involves a problem, as can be seen already in the spherical case: Consider, instead of V_s, the mass m of a spherically symmetric configuration as a function of the central pressure p_c. For degenerate matter $m(p_c)$ is well known

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