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Classification of Singular Sobolev Connections by Their Holonomy

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Abstract. For a connection on a principal $SU(2)$ bundle over a base space with a codimension two singular set, a limit holonomy condition is stated. In dimension four, finite action implies that the condition is satisfied and an a priori estimate holds which classifies the singularity in terms of holonomy. If there is no holonomy, then a codimension two removable singularity theorem is obtained.

1. Introduction

Since the appearance of the fundamental paper of Uhlenbeck [U_1], apparent point singularities in gauge field theories have been studied extensively and are now fairly well understood [O_1, OS, P, S, SS_1, Sm, U_1, U_2]. The next step in this development concerns singular sets of higher dimension. If the singular set has co-dimension three or higher, the techniques used for point singularities (subelliptic estimates and broken Hodge gauges) can still be applied [O_2]. Alternatively, these higher co-dimensional results, as well as the original removable point singularity theorems, can be obtained by straightforward modifications of the results in Sect. 5. On the other hand, co-dimension two singular sets, such as an S^2 embedded in S^4, introduce new geometric difficulties. These arise mainly from the fact that the complement of the singular set is no longer simply connected. Geometrically, this means that there are non-trivial *flat* connections. These connections cannot, in general, be extended to a neighborhood of the singular set even though the bundle itself may be topologically trivial. The appearance of holonomy is the obstacle to this extension. Physically, these singular sets give rise to finite action connections satisfying field equations whose topological charge is not integral but depends critically on holonomy [A, B, C, FHP_1, FHP_2].

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