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Global Stability of the Rarefaction Wave of a One-Dimensional Model System for Compressible Viscous Gas

Akitaka Matsumura[1] **and Kenji Nishihara**[2]

[1] Department of Mathematics, Kanazawa University, Kanazawa 920, Japan
[2] School of Political Science and Economics, Waseda University, Tokyo 169, Japan

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Abstract. This paper is concerned with the asymptotic behavior toward the rarefaction wave of the solution of a one-dimensional barotropic model system for compressible viscous gas. We assume that the initial data tend to constant states at $x = \pm\infty$, respectively, and the Riemann problem for the corresponding hyperbolic system admits a weak continuous rarefaction wave. If the adiabatic constant γ satisfies $1 \leqq \gamma \leqq 2$, then the solution is proved to tend to the rarefaction wave as $t \to \infty$ under no smallness conditions of both the difference of asymptotic values at $x = \pm\infty$ and the initial data. The proof is given by an elementary L^2-energy method.

1. Introduction

Subsequent to [10] and [11], we consider the Cauchy problem of a one-dimensional barotropic model system for compressible viscous gas. Our problem is described as

$$\begin{cases} v_t - u_x = 0 \\ u_t + p(v)_x = \mu\left(\dfrac{u_x}{v}\right)_x \\ p(v) = av^{-\gamma}, \quad x \in R, \quad t \in R_+ = (0, \infty) \end{cases} \tag{1.1}$$

with the initial data

$$(v, u)(0, x) = (v_0, u_0)(x), \tag{1.2}$$

where $v(>0)$ is the specific volume, u is the velocity, $\mu(>0)$ is the constant coefficient of viscosity and p is the pressure given by $p = av^{-\gamma}$ for a constant $a > 0$ and the adiabatic constant $\gamma \geqq 1$. We assume the initial data asymptotically tend to the