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The Equivalence of the Logarithmic Sobolev Inequality and the Dobrushin–Shlosman Mixing Condition*

Daniel W. Stroock[1] and Boguslaw Zegarlinski[1,2]

[1] 2-272, Mathematics Department, M.I.T., Cambridge MA 02139, USA
[2] Fakultät für Mathematik, Ruhr-Universität Bochum, Postfach 102 148, W-4630 Bochum 1, FRG

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Abstract. Given a finite range lattice gas with a compact, continuous spin space, it is shown (cf. Theorem 1.2) that a uniform logarithmic Sobolev inequality (cf. 1.4) holds if and only if the Dobrushin–Shlosman mixing condition (cf. 1.5) holds. As a consequence of our considerations, we also show (cf. Theorems 3.2 and 3.6) that these conditions are equivalent to a statement about the uniform rate at which the associated Glauber dynamics tends to equilibrium. In this same direction, we show (cf. Theorem 3.19) that these ideas lead to a surprisingly strong large deviation principle for the occupation time distribution of the Glauber dynamics.

0. Introduction

In our earlier article [S&Z], we showed (cf. Remark 3.23 in [S&Z]) that the Dobrushin–Shlosman mixing condition (cf. (1.5) below) guarantees that the corresponding (necessarily unique) Gibbs state satisfies a logarithmic Sobolev inequality (cf. (1.1) below) and asserted that we would be proving a converse statement in a forthcoming article. This is that "forthcoming article," and the promised converse is the content of Theorem 1.2 below. Because it is difficult to give precise statements of our results before we have introduced the notation explained in Sect. 1 below, we will confine our discussion in this introduction to a few general remarks of a somewhat historical nature. In particular, all that we hope to explain here is the general principle on which our analysis rests.

Ever since the ground-breaking work of Dobrushin, Lanford and Ruelle, most of the analysis of Ising-type models has concentrated on the associated Gibbs

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