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The Quantum Strip: Liouville Theory for Open Strings

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Abstract. The quantum group structure of 2D gravity recently put forward by one of us (J.-L. G.) is used to study quantum gravity on the strip. The boundary conditions, previously studied by A. Neveu and this author become easy to implement when one introduces the universal family of chiral operators associated with $U_q(sl(2))$. A general formula for inverse powers of the metric-tensor operator is thereby derived. It contains a new universal matrix A, acting in representation-space, which obeys identities involving the R matrix, the Clebsch–Gordon coefficients, and the co-products of $U_q(sl(2))$. The physical meaning of these identities is to ensure that these powers of the metric are local and closed by fusion.

1. Introduction

In the early days, Gervais and Neveu developed their operator-approach to Liouville theory by dealing mostly with open surfaces [1–6], and imposing classical boundary conditions derived from studies of the Weyl anomaly on surfaces with boundaries [7, 8]. Since then, the algebraic approach initiated in refs. 3, 4 has made significant progress when its connection with quantum groups was recognized [9–13], but in recent times, 2D gravity with boundaries has received little attention apart from ref. 14. It is the aim of the present paper to go back to this problem. One will see that the quantum-group structure of refs. 9–13 leads to considerable simplifications and progress. Indeed, although Gervais and Neveu completely solved the classical problem on the strip [1, 2], they could only derive the quantum expression for the simplest inverse power of the metric tensor [4], obtaining a rather complicated expression by imposing locality on the world-sheet. From the

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