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Analyticity of the Scattering Operator for the Nonlinear Klein–Gordon Equation with Cubic Nonlinearity

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Abstract. The wave and scattering operators for the equation

$$(\Box + m^2)\varphi + \lambda\varphi^3 = 0$$

with $m > 0$ and $\lambda > 0$ on four-dimensional Minkowski space are analytic on the space of finite-energy Cauchy data, i.e. $L_2^1(\mathbf{R}^3) \oplus L_2(\mathbf{R}^3)$.

1. Introduction

This paper answers a question, raised by Baez and Zhou in [2], whether the scattering and wave operators for

$$(\Box + m^2)\varphi + \lambda\varphi^3 = 0, \quad m > 0, \quad \lambda > 0, \tag{1}$$

are analytic or not on the whole space of finite-energy data $L_2^1(\mathbf{R}^3) \oplus L_2(\mathbf{R}^3)$. The answer is affirmative. This implies, which is noted in [2], that the massive φ^4 theory is completely integrable. The same holds for the massless φ^4 theory proved by Baez in [1].

We start by introducing some notation and some basic facts following the presentation given in [2]. Consider Eq. (1), where $\varphi = \varphi(t, x)$ is a realvalued function on $\mathbf{R} \times \mathbf{R}^3$. Let $L_q^s(\mathbf{R}^3)$ denote the Sobolev space of functions on $\mathbf{R}^3$ with s derivatives in L_q and let $\mathbf{X}$ denote the Hilbert space $L_2^1(\mathbf{R}^3) \oplus L_2(\mathbf{R}^3)$ with norm $\|\cdot\|_{\mathbf{X}}$ given by

$$\|(u_1, u_2)\|_{\mathbf{X}} = \left(\frac{1}{2}\int_{\mathbf{R}^3} (|\nabla u_1|^2 + m^2 u_1^2 + u_2^2)dx\right)^{1/2}.$$

Given $u \in \mathbf{X}$ there is a unique distributional solution φ of (1) with

$$(\varphi, \dot{\varphi})|_{t=0} = u. \tag{2}$$

Let $U(t)u = (\varphi, \dot{\varphi})|_t$ and let $U_0(t)$ be the orthogonal linear operator on $\mathbf{X}$ corresponding to the case $\lambda = 0$, i.e. the linear Klein–Gordon equation. If $N(u_1, u_2) =$