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On Multivortices in the Electroweak Theory II: Existence of Bogomol'nyi Solutions in $\mathbb{R}^2$

Joel Spruck[1]* and Yisong Yang[2]

[1] Department of Mathematics, University of Massachusetts, Amherst, MA 01003, USA
[2] Department of Mathematics and Statistics, University of New Mexico, Albuquerque, NM 87131, USA

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Abstract. In this paper we study the Bogomol'nyi equations of the electroweak theory in the full plane. We will show that, for any distribution of the vortices, there exists a two parameter family of gauge-distinct solutions. Moreover, we also establish some sharp decay rate estimates for these solutions.

1. Introduction

In Part I of this paper [7], we have proven the existence of Abrikosov like periodic vortices in the bosonic sector model proposed by Ambjorn and Olesen [3, 4] of the Glashow–Salam–Weinberg theory. These solutions were found from a Bogomol'nyi system of first order equations which take on a more complicated form than in the classical abelian case due to the anti-screening of the magnetic field. As a result, this system further reduces to a semilinear elliptic system of nonstandard type and we showed in Part I that the number of such vortices is bounded above in terms of the relevant physical parameters, although the locations may be prescribed arbitrarily.

The goal of the present paper is to study this Bogomol'nyi system for the self-dual electroweak interactions in the full space $\mathbb{R}^2$. These solutions are necessarily of infinite energy and thus the method of Part I cannot be directly applied. Our main strategy then, is to combine the method of weighted Sobolev spaces, used by McOwen [6] in his study of conformal deformation equations, with the crucial change of variables introduced in Part I to reduce our elliptic system to a lower diagonal form. As a result, we are able to show (Theorem 3.3) that for any distribution of vortex locations there is a two parameter family of

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