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On Holomorphic Factorization of WZW and Coset Models

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Abstract. It is shown how coupling to gauge fields can be used to explain the basic facts concerning holomorphic factorization of the WZW model of two dimensional conformal field theory, which previously have been understood primarily by using conformal field theory Ward identities. We also consider in a similar vein the holomorphic factorization of G/H coset models. We discuss the G/G model as a topological field theory and comment on a conjecture by Spiegelglas.

1. Introduction

The WZW model of two dimensional conformal field theory [1] is a quantum field theory in which the basic field is a map $g:\Sigma\to G$, Σ being a two dimensional Riemann surface and G being a compact Lie group, which for simplicity we will in this paper take to be simple, connected and simply connected. The basic WZW functional is

$$I(g)=-\frac{1}{8\pi}\int_{\Sigma} d^2\sigma\sqrt{\varrho}\varrho^{ij}\,\mathrm{Tr}(g^{-1}\partial_i g\cdot g^{-1}\partial_j g)-i\Gamma(g), \tag{1.1}$$

where ϱ is a metric on Σ, Tr is an invariant form on the Lie algebra of G whose normalization will be specified presently, and Γ is the Wess-Zumino term [2]. The latter has the following description [3] in case Σ is a Riemann surface without boundary. (For the more general case see [4].) Let B be a three manifold such that $\partial B=\Sigma$, pick an extension of g over B, which we will also call g, and let

$$\Gamma(g)=\int_B g^*\omega=\frac{1}{12\pi}\int_B d^3\sigma\varepsilon^{ijk}\,\mathrm{Tr}\,g^{-1}\partial_i g\cdot g^{-1}\partial_j g\cdot g^{-1}\partial_k g, \tag{1.2}$$

where ω is the left and right invariant three form on the G manifold defined by

$$\omega=\frac{1}{12\pi}\mathrm{Tr}(g^{-1}dg\wedge g^{-1}dg\wedge g^{-1}dg). \tag{1.3}$$

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