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Smoothing Properties and Retarded Estimates for Some Dispersive Evolution Equations

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Abstract. Smoothing properties, in the form of space-time integrability properties, play an important role in the study of dispersive evolution equations. A number of them follow from a combination of general arguments and specific estimates. We present a general formulation which makes the separation between the two types of ingredients as clear as possible, and we illustrate it with the examples of the Schrödinger equation, of the wave equation, and of a class of 1+1 dimensional equations related to the Benjamin-Ono equation. Of special interest for the Cauchy problem are retarded estimates expressed in terms of those properties. We derive a number of such estimates associated with the last example, and we mention briefly an application of those estimates to the Cauchy problem for the generalized Benjamin-Ono equation.

1. Introduction

A large amount of work has been devoted in the last twenty years to the space-time integrability properties of solutions and to the smoothing properties of dispersive partial differential equations of the type

$$\partial_t u - Lu = f\,, \tag{1.1}$$

where u is the unknown (possibly vector valued) function, defined in space-time $\mathbb{R}^{n+1}$, L is a skew adjoint operator in some Hilbert space $\mathscr{H}$ of functions of the space variable, for instance $\mathscr{H} = L^2(\mathbb{R}^n)$, and f can be a given external source term, or a (possibly non-linear) function of the unknown function u, or a combination of both [3–10, 12–24]. Let $U(t) = \exp(tL)$ be the unitary one parameter group in $\mathscr{H}$ generated by L. The Cauchy problem for (1.1) with initial data $u(t=0) = u_0$ is formally equivalent to the integral equation

$$u(t) = U(t)u_0 + \int_0^t d\tau\, U(t-\tau)f(\tau) \tag{1.2}$$

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