

Dynamical Entropy of Quasi-local Algebras in Quantum Statistical Mechanics[★]

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Abstract. We study the dynamical entropy in the sense of Connes, Narnhofer, and Thirring of automorphisms on quasi-local algebras in quantum statistical mechanics. We extend their Kolmogorov-Sinai type theorem for AF-algebras to quasi-local algebras which are not necessarily AF-algebras.

1. Introduction

In their recent paper [5] Connes, Narnhofer and Thirring generalized the notion of a dynamical entropy introduced by Kolmogorov and Sinai [8, 15] for classical dynamical systems to the case of automorphisms of C^* -algebras invariant with respect to a given state, and they obtained a Kolmogorov-Sinai type theorem [5] for approximately finite (AF) algebras. The main purpose in this paper is to extend their result to quasi-local algebras in quantum statistical mechanics which are not necessarily approximately finite.

The concept of the Kolmogorov-Sinai (KS) entropy of measure preserving transformations became a key notion in ergodic theory [2] and also it allowed a formulation of the variational principle in statistical mechanics [14]. A quantum or non-commutative analogue of KS entropy was required for both to provide an important mathematical concept for quantum dynamical systems and to be applicable in quantum statistical mechanics. There have been several attempts to generalize the classical theory to non-commutative cases [3, 4, 6, 10]. In order that a generalization is meaningful, it is natural to require that if it is restricted to classical systems, it must reduced to the KS entropy and that it has the continuity of the KS type [8, 15] to be able to compute the entropy. Due to much progress in quantum statistical mechanics [1, 9], Connes, and Störmer were able to handle the

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