On the Symplectic Geometry
of the Super Teichmüller Space*

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Abstract. The geometry of the Teichmüller space of the super Riemann surfaces
is examined. The Weil–Petersson Kähler form is calculated in terms of the super
coordinate functions which provide local coordinates for the super Teichmüller
space. It is shown that the Kähler form on the super Teichmüller space is closed.

1. Introduction

The purpose of this paper is to show that the Weil–Petersson Kähler form on the
super Teichmüller space for the compact super Riemann surfaces of genus $h \geq 2$ is
closed. As for the ordinary (non-super) Riemann surfaces, it is known that the
Teichmüller space is a complex Kähler manifold, and hence the Kähler form is
closed. A Kähler metric, defined in terms of the Petersson product for the auto-
morphic forms, was introduced by Weil [1]. The Weil–Petersson metric may
naturally project to the moduli space because it is invariant under the covering
transformations.

Wolpert [2] gave a description of the Fenchel–Nielsen deformation [3] in
terms of quasiconformal mappings. The Fenchel–Nielsen vector fields t_*, which
are generators of the deformation, were found to be related to the geodesic length
functions l_*, introduced by Fricke and Klein [4] to provide local coordinates for
the Teichmüller space. He showed a duality formula,

$$\omega(t_*) = -dl_*, \tag{1}$$