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The Unitary Irreducible Representations of the Quantum Heisenberg Algebra

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Abstract. The goal of this work is to describe the irreducible representations of the quantum Heisenberg algebra and the unitary irreducible representation of one of its real forms. The solution of this problem is obtained through the investigation of the *left spectrum* of the quantum Heisenberg algebra using the results about spectra of generic algebras of skew differential operators (cf. [R]).

Introduction

The *quantum Heisenberg* algebra $H_{q,h}(k) = H_{q,h}(x, y|k)$ is the algebra over the field k generated by indeterminables x, y satisfying the relation

$$xy - qyx = h,$$

where $q, h \in k^*$. This algebra (or, rather this family of algebras) was introduced in [FG] and, it seems, is going to occupy in the quantum groups – the quantum spaces realm (cf. [Dr, J, FRT, M, S]) nearly the same place as its "classical limit" – Weyl algebra – does in representation theory (cf. [K, D]) and mathematical physics. Consequently, it deserves a thorough investigation.

The object of this work is to obtain a description of irreducible representations of the quantum Heisenberg algebra and unitary irreducible representation of one of its real forms. We come to the solution of this problem through the investigation of the *left spectrum* of $H_{q,h}(k)$. Recall the definition of the left spectrum.

Let $\succ$ be the following relation (preorder) on the set $I_l R$ of left ideals of an associative ring R: $m \succ n$ if there exists a finite subset $w \subset R$ such that the left ideal $(m:w) := \{r \in R | rw \subset m\}$ is contained in n. The left spectrum, $\mathrm{Spec}_l R$, of the ring R consists of all the left ideals p of R such that $(p:x) \succ p$ for every $x \in R - p$. We use mostly the following two properties of $\mathrm{Spec}_l R$: 1) $\mathrm{Spec}_l R$ contains the set $\mathrm{Max}_l R$ of maximal left ideals of R; 2) the left spectrum is preserved by localizations (as well as left maximal ideals).

The results obtained in [R] about the left spectrum of the general algebra of skew differential operators $[A[x; \vartheta, d]$ are sufficient to get a complete description