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Conformal Blocks of Minimal Models on a Riemann Surface

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Abstract. We give explicit integral representations for conformal blocks of minimal models on arbitrary compact Riemann surfaces.

1. Introduction

Rational conformal field theory on the Riemann sphere S can be formulated in terms of a pair $\mathscr{L} \oplus \mathscr{L}$ of identical Virasoro algebras $\mathscr{L}$ with assigned rational central charge c. If $L(h, c)$ denotes the irreducible highest weight $\mathscr{L}$-module of highest weight h, the Hilbert space of the theory decomposes as $\bigoplus_{h,h'} L(h, c) \otimes L(h', c)$, with h and h' ranging in some finite set of rational values. Correlation functions of local fields at point $P_1, \ldots, P_n$ on S admit an analogous decomposition into the so-called left (respectively right) conformal blocks, which depend holomorphically (respectively antiholomorphically) on the local coordinates $z(P_i)$ defined about the P_i.

Under certain consistency requirements, the theory can be generalized to Riemann surfaces Σ of positive genus. The main argument for this lies in a formalization of the surgery operations ("sewing") through which Σ can be obtained from a set of three-punctured spheres. General formulations of conformal field theory on Riemann surfaces have been outlined by Segal and by Gawedzki. They can be roughly summarized as follows: The holomorphic part of a conformal field theory is specified by assignments $\Sigma \mapsto B(\Sigma)$ of objects B to Riemann surfaces. If Σ has $m+n$ punctures at points P_i, the $B(\Sigma)$: $\bigotimes_{1 \leq i \leq m} L(h_i, c) \to \bigotimes_{m+1 \leq i \leq m+n} L(h_i, c)$ are trace class operators depending holomorphically on $z(P_i)$ and having specified properties under conformal diffeomorphisms. Moreover, the assignment B from

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