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On Multivortices in the Electroweak Theory I: Existence of Periodic Solutions

Joel Spruck[1]* and Yisong Yang[2]

[1] Department of Mathematics, University of Massachusetts, Amherst, MA 01003, USA
[2] Department of Mathematics and Statistics, University of New Mexico, Albuquerque, NM 87131, USA

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Abstract. In this paper we consider the bosonic sector of the electroweak theory. It has been shown in the work of Ambjorn and Olesen that when the Higgs mass equals to the mass of the Z boson, the model in two dimensions subject to the 't Hooft periodic boundary condition may be reduced to a Bogomol'nyi system and that the solutions of the system are vortices in a "dual superconductor". We shall prove using a constrained variational reformulation of the problem the existence of such vortices. Our conditions for the existence of solutions are necessary and sufficient when the vortex number $N = 1, 2$.

1. Introduction

Instantons, monopoles, and vortices form a rich spectrum of topologically elegant solutions of gauge field theories. Vortices arise in two-dimensional models in which the gauge symmetry is spontaneously broken via Higgs bosons. Such solutions represent string-like field configurations in higher dimensions and, in the context of the abelian Higgs theory, were first discovered in Abrikosov's poineering study [1] of the magnetic properties of superconducting materials. In recent years, due to their interesting roles in grand unified theories, especially in cosmology [10], nonabelian vortices have attracted a considerable amount of attention. It is well-known that one of the most important and successful nonabelian gauge field theories is the electroweak theory of Glashow, Salam and Weinberg, where the gauge group is $SU(2) \times U(1)$. In a series of papers, Ambjorn and Olesen [3–5] proposed that a class of periodic vortex-like solutions similar to those of Abrikosov occur in this electroweak theory (see also Skalozub [11, 12]). They showed that,

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